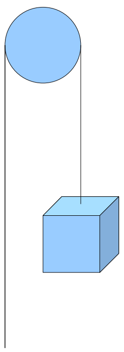
Gears

1. Although this subject is not examinable as part of the Senior Cadet course in Propulsion, it is of relevance to any cyclist or car driver.

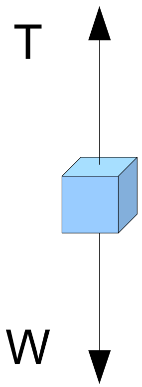
The crankshaft of an internal combustion engine turns at a speed of at least 1,000 RPM at all times, yet the wheels of the car clearly do not spin as fast. It's also possible for the engine to be running, but for the car to be stationary. The reason for this is that the crankshaft does not feed rotational power directly to the wheels of the car, but does so through a gear box and clutch.

The gearing mechanism on a car or bicycle is intended to convert the rotational force of the engine into a form in which it is most useful, trading off power and speed. Newton's second law tells us that force is proportional to acceleration. Therefore, when we first move off and whenever we accelerate to overtake another vehicle, we need significant rotational force (torque) from the engine. To maintain a constant high speed, however, the force of the engine needs only to be sufficient to counteract the drag forces in the car. Force is proportional to acceleration, and constant speed means no acceleration. To drive on a motorway, then, the engine needs to be rotating fast, but we don't need much torque.

Dealing with rotating forces is a complex field of mechanics – in fact it's largely ignored even at A-Level Mathematics and not introduced in full until Further Mathematics. However, there is a type of gear that is simpler to understand that will help to explain what we set out to achieve with car or bicycle gears: the pulley.



2. The diagram to the left shows a heavy object suspended from one side of the pulley, with the free end of the rope passing over the pulley. The question is: how hard do I need to pull on the free end of the rope in order to raise the pulley? That is, what force must I apply in order not to let the object drop to the ground?

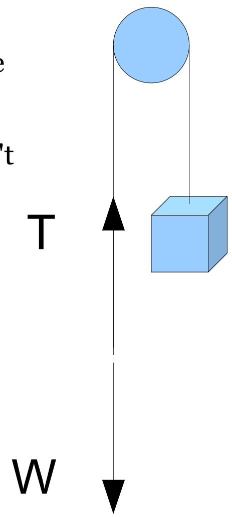


3. There are two forces acting on the heavy object: weight (the force of gravity acting on the mass) and the tension in the rope. By Newton's first law, if the block isn't accelerating, then the forces must be equal and opposite.

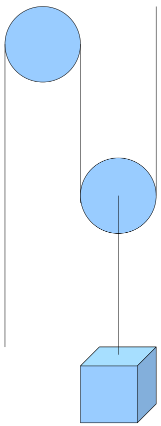
$$\mathbf{W = T}$$

or, more accurately,

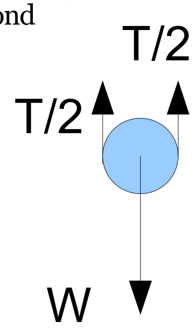
$$\mathbf{W = -T}$$



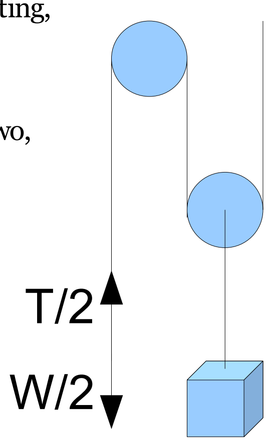
4. The tension in the rope is the same throughout. Therefore, the force I need to hold the free end of the rope is equal to the tension. This stands to reason: if we hang identical objects on each end of the rope, we would expect them to balance each other.



5. The diagram to the left shows a two-pulley system. The same heavy object is attached to the second pulley.

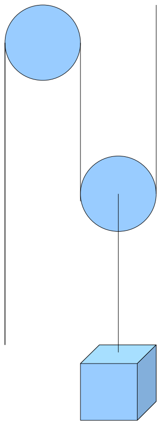


6. The object still isn't accelerating, so the total downwards force (weight) must match the total upwards force. However, the upwards force is now split in two, each half the size of before. In other words, the tension in the rope is half as much as it was before.

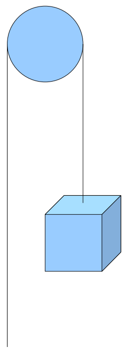


7. Tension in the rope is constant throughout the rope. So, to balance the weight of the object using two pulleys, I only need half the amount of force as with one pulley.

Gears

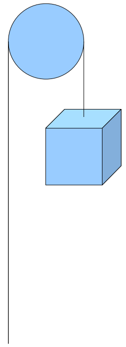


8. If I add more pulleys, the force I need apply drops proportionately. However, for the purposes of this demonstration, we will stick with two pulleys.
Now we will look at what work needs to be done in order actually to lift the object with one pulley.
Work has many definitions, all equivalent. The relevant one here is: *work is the amount of force applied multiplied by the distance moved.*



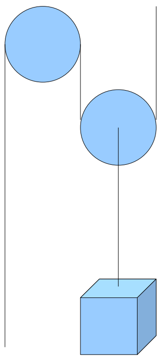
9. We've already seen that, with one pulley, the amount of force needed to balance the weight of the object is an equal force. Ignoring friction and air resistance, any net increase in the force on the free end of the rope will start the object moving upwards.
To raise the object by 1 metre, I need to pull the rope with a force of W for 1 metre.

Work = W x 1m

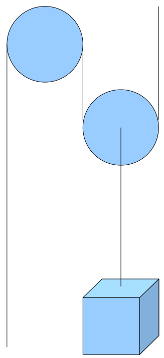


10. Another measure of the work done is the change in energy of the system as a whole. Energy can never be created or destroyed, merely changed from one form to another. In this case, having raised the object, it has gained *gravitational potential energy* equal to the work done on it.
The **power** applied to the system is a measure of the *rate of change of energy*, or *work done per unit time*. If it took me 1 second to haul the rope 1m, then the power I applied to the system is

Power = (W x 1m) / 1s



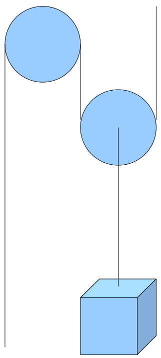
11. However, in the two-pulley case, if I pull the free end of the rope 1m, the object does not rise by 1m. The total length of the rope shortens by 1m, but the object only rises by half this amount. If I want to raise the object by the full 1m, I must pull in 2m of rope.



12. Although I have to pull the rope twice as far, remember that I only needed to use half the force on the rope: W/2. The **work done** is still *force times distance.*

Work = W/2 x 2m
= W x 1m

So, the total amount of work done is exactly the same. The *gravitational potential energy* gained by the object is still also W x 1m.



13. But I might have a physical limit to how fast I can pull in the rope. If I can only haul in 1m of rope in a second, then it will take me 2s to pull in all the rope. The **power** is therefore halved:

Power = (W/2 x 2m) / 2s

14. Now consider the question the other way around. Instead of varying the strength with which I pull on a rope attached to a constant object, I could have a system that has a fixed maximum strength to pull the rope: in which case, what is the maximum load I can lift?

This situation is much like a car engine. Revving the engine gives us a certain maximum amount of rotational force (torque) and it's up to us to use it as appropriate.

Gears therefore provide us with a trade-off. To pull off and accelerate, we add extra "pulleys" - that is, we use a lower gear – which helps us overcome the heavy "load". But we can't run the engine faster and faster: there's that physical limit on how fast I can pull in the rope or, equivalently, at very high speeds, an engine might explode. So, once we've reached the speed we want and don't need to accelerate any more, our "load" is lighter (remember, force is proportional to acceleration) and we can get away with using fewer "pulleys" or a higher gear.